

A Vopěnka-style principle for fuzzy mathematics

Libor Běhounek

University of Ostrava

The indistinguishability of objects can be mathematically modeled in many different ways—e.g., using classical equivalence or proximity relations, via metrics or topology, using rough sets, or using fuzzy similarity relations. Modeling indistinguishability by means of fuzzy similarity relations (or fuzzy equivalences, [1]) is elegant in that it provides a solution to Poincaré’s paradox [2]—namely, the contradiction consisting in the fact that indistinguishability should intuitively be transitive, yet in a sufficiently long series whose every two neighboring elements are mutually indistinguishable, the extremal elements may be distinguished: fuzzy equivalences allow the latter, but are still transitive in the sense of fuzzy logic. Fuzzy equivalences happen to be dual to (generalized) metrics, so many metric and topological notions carry over to fuzzy equivalences.

Vopěnka’s Alternative Set Theory (AST, [3]) has its own intriguing model of indistinguishability, construed as a non-standard equivalence relation arising by discrimination via progressively sharpened perspectives towards the horizon. Here, however, I will draw on another fundamental idea of Vopěnka’s AST, namely his characterization of finite sets in terms of the surveyability and clear discernibility of all of their elements by the limited human means (even if idealized). One way of interpreting Vopěnka’s principle of infinity-as-indiscernibility is that in any infinite set, some elements are inevitably indistinguishable from each other.

If we abstract away from the specifics of AST and apply the latter principle to the model of indistinguishability in fuzzy mathematics, it amounts to the requirement of (metric) precompactness, or the total boundedness of the generalized metric dual to the fuzzy indistinguishability relation. This requirement can be easily expressed by means of formal fuzzy logic and investigated by the methods of formal fuzzy mathematics. In the talk, I will show some consequences of this principle, such as the existence of fuzzy minima in fuzzy orderings compatible with a precompact fuzzy equivalence relation,

and the use of this general fact in the recently proposed fuzzy semantics of counterfactual conditionals [4].

References

- [1] Llorenç Valverde. On the structure of F -indistinguishability operators. *Fuzzy Sets and Systems*, 17(3):313–328, 1985.
- [2] Henri Poincaré. *La Science et l'Hypothèse*. Flammarion, Paris, 1902.
- [3] Petr Vopěnka. *Mathematics in the Alternative Set Theory*. Teubner, Leipzig, 1979.
- [4] Libor Běhounek and Ondrej Majer. A graded semantics for counterfactuals. *Synthese*, 199:11963–11994, 2021.