

A hitherto unknown text by Bolzano on his *Beyträge*

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The background

Bolzano was born in the midst of a period of reforms, including:

- Extension of philosophical studies from 2 to 3 years.
- Teaching in German in institutions above elementary level.
- Emphasis on the teaching of mathematics: useful for the development and improvement of intellect, other sciences and everyday life.

The philosophical studies (1796-99) included:

- 3 courses in mathematics: “one cannot be content to accept propositions as proven for which only inadequate and to some extent incorrect inferences have been presented” (Kästner 1758).
- 1 course in logic...: deals “with the thinking in general, the development of the concept of truth and the study of the nature of error” (Feder 1778).

Considerations on Some Objects of Elementary Geometry (1804).

- One must seek the proof of every proposition despite its obviousness and must not accept proofs that use *alien* concepts to the thesis to be proved.

Examination contests for Vydra's chair of Elementary Mathematics and for one of the new chairs of Religious Doctrine.

- He displayed "his talent" by a "thorough approach and erudition."

"One can make a science worthwhile in three ways: first, by making its presentation easier for the students' understanding; second, by enlarging the science through new discoveries; third, by developing the principles of the science for the reason. / Following Prof. Gerstner's advice, I have decided not to work publicly on the third subject." (MM)

The book

Contributions to a Better-Grounded Presentation of Mathematics (1810)

Preface: “the first foundations of this (otherwise so magnificent) structure are still not completely secure and in order.” (Russ 2004)

1st part: He proposes a new definition of mathematics (“deals with the general *laws* (forms) to which things must conform in their existence”) and a new division of its disciplines.

2nd part: Remarks on mathematical –scientific knowledge– method, i.e., logic.

Appendix: “On the Kantian Theory of the Construction of Concepts through Intuitions”

Temporal aetiology
(sensible things in time)

Mechanics
(... in time and space)

(unfree sensible things *in concreto*)

Chronometry
(theory of time)

Geometry
(theory of space)

(unfree sensible things *in abstracto*)

Aetiology
(theory of grounds)

General mathesis
(things in general)

Arithmetic, algebra, analysis, theory of combinations

There is an “objective connection” (i.e., “independent of our [...] subjective recognition”) among “true judgements or truths”: some are grounds or consequences of others.

- Judgements are the propositions that constitute inferences and have the subject-predicate form.
- He does not distinguish between the mental act of a judgement and its content.

He addresses the different parts of the mathematical apparatus (concepts, circumscriptions, axioms, etc.).

He lists 5 forms of judgements according to their copula:

- Of necessity: A is a kind of B .
- Of possibility: A can be a kind of B .
- Of duty: the free rational being N should do X .
- Of perception or empirical: I perceive X .
- Of probability.

The manuscript

Bolzano's written estate held at the Literary Archives of the Museum of National Literatur includes his transcriptions of reviews on his *Considerations* (BG) and *Contributions* (BD):

1	<i>Neue Leipziger Literaturzeitung, 1805</i>	BG
2	<i>Allgemeine Literatur-Zeitung, 1806</i>	BG
3	<i>Heidelbergische Jahrbücher der Literatur, 1808</i>	BG
4	?	BD
	<i>Heidelbergische Jahrbücher der Literatur, 1810</i>	BD
5	<i>Annalen der Literatur und Kunst in dem Oesterreichischen Kaiserthume, 1811</i>	BD, BG

Josef Fesl (Bolzano's pupil and friend) wanted Josef Jenko (professor of math in Vienna) to take more interest in Bolzano's math thinking (letter, 1842).

- Fesl preserved Jenko's written estate and bequeathed all his property to the National Museum, from where Jenko's written estate was transferred to the Literary Archives, being reclassified in 1971.

Bolzano → Fesl → Jenko → Fesl → NM → LA PNP → LA PNP*

Reference of each review in the transcriptions, with the exception of the 4th.

- Within *Bernard Bolzano–Gesamtausgabe* all manuscripts appear in the “Katalog” of LA PNP but not in “Bolzano-Gesamtbibliographie 1804–1999”.

The announcement

The manuscript is a **draft of an announcement** published on the 9th of January 1811 in the *Allgemeine Literatur-Zeitung*.

um auf den religiösen Sinn unseres Zeitalters wohlthätig zu wirken, den Weg, den er zu betreten hat, um sich bey dem gegenwärtigen Zustande der Wissenschaften (namentlich der theologischen) am vortheilhaftesten für jenen heiligen Beruf zu bilden, und die in unsern Tagen vorzüglich zu beachtende Aufrechthaltung und Feststellung seines äußern Ansehens und seiner bürgerlichen Verhältnisse betreffen soll; theils aus kurzen *Anzeigen*, *Nachrichten* und *Anfragen*, welche mit jenem Endzwecke in Verbindung stehen (Erfahrungen von den Schwierigkeiten, aber auch von der wohlthätigen Wirksamkeit des Predigtamtes, Nachrichten von den Einrichtungen und Anordnungen, welche Kirchen und Schulen betreffen, Vorschläge zu Verbesserungen)

A. L. Z. 1811. Erster Band.

II. Ankündigungen neuer Bücher.

Bolzano, B., Beyträge zu einer begründeteren Darstellung der Mathematik. Erste Lieferung. 8. Prag 1810. Widtmann. 16 gr.

Mit dieser, durch Versehen in dem Leipziger Messcataloge nicht angezeigten, Schrift beginnt der Verfasser, ein durch tiefes Studium der Mathematik ausgearbeiteter, philosophischer Geist, eine Reihe von Untersuchungen, durch welche, unserm Dafürhalten nach, eine *neue Epoche nicht in der Mathematik allein, sondern wohl überhaupt in allen speculativen Wissenschaften* veranlaßt werden dürfte. Denn die Verbesserungen des

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The *Allgemeine Literatur-Zeitung* (Halle) included an *Intelligenzblatt*.

- Announcement: unsigned, published by Widtmann (the book's publisher).
- Prague publishers rarely published in *ALZ Intelligenz* at the time: this is the only one by Widtmann in 1808-12 and is longer than usual.

The book “was not advertised in the Leipzig Fair Catalogue due to an **oversight**”.

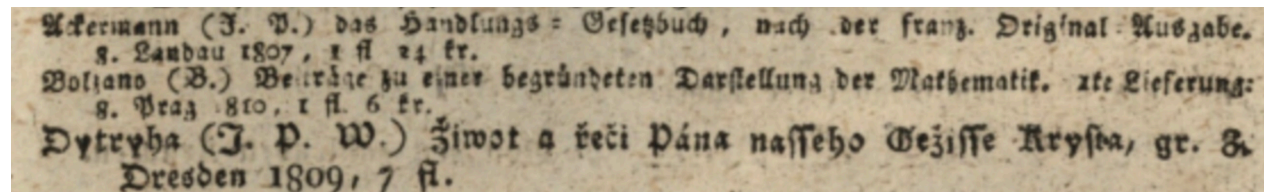
- The catalogue reported on the publications available at the Easter or St. Michael bookfairs in Frankfurt and Leipzig (printed 1 week prior to fairs).

It was listed in the 1811 Easter catalogue, which followed the announcement.

- The 1810 Easter catalogue includes works by Widtmann, not St. Michael's.

Goethe got a copy of the *Beyträge* in July or August 1810 (Künne 2012).

An announcement of “new books” by the Brno and Olomuc bookseller and publisher Johann Georg Gastl includes it (*Brünner Zeitung*, June 30, 1810).



The *Catalogue of the New Books which came out at the last Easter Fairs in Frankfurt and Leipzig*, by the Erlangen bs. and p. Johann Jacob Palm, includes it.

**Bolzano, B., Beiträge zu einer begründeteren
Darstellung der Mathematik, 1te Liefer. 8.
Prag, Widtmann 1 fl 12 fr**

The “oversight” seems to correspond to the Easter catalogue (7-13 May); the manuscript would date from between 13 May (30 September?) and 9 January.

The reception

As far as *method* is concerned I have generally kept to the principles set out in the *Beyträge* [...], because I am still convinced of their correctness. (1816/2004)

[T]he first [of the instalments of the *Beyträge*] had the misfortune, despite the importance of its contents, not even to be announced and reviewed in some scholarly journals, and in others only very superficially. (1817/2004)

1810: Bolzano's definition of mathematics is a "vague expression", his division of mathematics is set on an "not entirely apt basis of division", his "indeterminacy of thought [...] merely leads to general logical discussions" on math method.

- Due to his "misunderstanding" of Kantian theory of the construction of concepts, he "misjudges the intuitive nature of mathematical knowledge".

1811: the second part "contains much that is good" and is "far more satisfactory" than the first one, but, as the rest of the book, it "does not present anything better-grounded than has already been presented by others".

- Discusses Kant's theory "hardly to the satisfaction of the connoisseurs".
- "Thanks and encourage[s]" the "**modest author**", whose further investigations "should not be without benefit for the science".

1st: With BD “a **new epoch** is likely to be initiated” in mathematics “because [of] the author’s improvements”; “He reveals to us flaws [...] which had never been noticed before and, we believe, he has succeeded in removing them and in deriving the most important theorems from all parts of mathematics [...] from their proper and objective grounds.”

2nd: Description of the work. “[An] entirely new light is thrown upon the distinction between analytic and synthetic judgments.” “The author conceives the concept of a scientific proof more clearly than has ever been done before and is thereby led to a number of most important conclusions.”

3rd: This “shall suffice to encourage mathematicians as well as philosophers to a careful examination [...] of this work which, albeit small, is very rich in new and fruitful points of view. The publisher only adds that the author intends to publish a *Textbook of geometry* in accordance with his method in the near future.”

Final remarks

The texts enhance our knowledge of the book; the reviews, of how it was read.

“[Bolzano] must be considered one of the greatest logicians of all time [...]. Logic as a science must be based upon Bolzano’s work.” (Husserl 1900)

“[*Beyträge*] must be one of the first books devoted to what we would now call foundations of mathematics, or philosophy of mathematics.” (Russ 2004)

“[O]ffers an important contribution to Kant-scholarship –and, regrettably, one that remains largely unknown to Kant specialists.” (Lapointe 2011)

“The doctrine of the objective connection of truths is the seed from which all of Bolzano’s logical theories will grow.” (Rusnock & Šebestík 2013)

Moc Vám děkuji!