First-Order Logic of Questions

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In my talk I will present an overview of a research area known as inquisitive semantics (Ciardelli, 2016; Ciardelli et al., 2019; Grilletti, 2020; Punčochář, 2016, 2019). I will explain philosophical motivations behind this framework and its basic mathematical features. The focus will be mainly on the first-order version of the theory. The main results and open questions in this area will be presented.

Inquisitive semantics is a framework that allows us to represent questions and statements in a uniform way. This would not be possible in the standard semantics based on the notion of truth. Unlike statements, questions are not true or false. For this reason inquisitive semantics employs an "information-based" semantics in which logical connectives are not characterized by truth conditions but rather in terms of informational support. While truth can be captured as a relation between first-order models and formulas, support is a relation between information states and formulas. An information state can be intuitively conceived of as a (typically incomplete) representation of a structure. Formally, it can be defined as a set of first-order models—those models that are compatible with the representation. (We restrict ourselves to the cases where the models forming an information states share a common domain.) Hence, support is formally defined as a relation between sets of models and formulas (with respect to an evaluation of variables).

Let us define the basic framework more precisely. For the sake of simplicity, we just consider a standard first-order language without functional symbols and identity. As the basic logical symbols we can take $\bot, \wedge, \rightarrow, \forall$. The symbols $\neg, \vee$ and $\exists$ can be defined in terms of the basic symbols in the usual way.

Let s be a set of first-order models with a common domain U. An evaluation in s is a function that assigns to every variable an element from U. If e is an evaluation, x a variable and o an element from U, then $e(o/x)$ will denote, as expected, the evaluation that assigns o to x and $e(y)$ to every other variable y. The relation of support is then defined as follows:

$s \vDash_e Pt_1, \ldots, t_n$ iff for all $\mathcal{M} \in s$, $Pt_1, \ldots, t_n$ is true in $\mathcal{M}$ w.r.t. e,

$s \vDash_e \bot$ iff s is empty,

$s \vDash_e \varphi \wedge \psi$ iff $s \vDash_e \varphi$ and $s \vDash_e \psi$,

$s \vDash_e \varphi \rightarrow \psi$ iff for every $t \subseteq s$, if $t \vDash_e \varphi$, then $t \vDash_e \psi$,

$s \vDash_e \forall x \varphi$ iff for every $o \in U$, $s \vDash_{e(o/x)} \varphi$.

It can be easily shown that for every formula φ of this language, φ is supported by a state s (w.r.t. e) iff φ is true in every model of s (w.r.t. e) in the sense of the standard semantics for classical logic. As a consequence, for the basic language, the logic determined by this semantics based on support conditions coincides with classical first-order logic. However, the merit of this setting is that it allows us to extend the language with questions and equip them with a suitable semantics. Questions are introduced into the language via two question-forming operators: inquisitive disjunction $\vee\!\!\!\vee$ and inquisitive existential quantifier $\exists\!\!\!\exists$. For example,

- $Pa \mathbin{\vee\mkern-13mu\vee} Qa$ represents the question *whether* a *has the property* P *or the property* Q,
- $\exists\mkern-11mu\exists xPx$ represents the question that asks *what is an object that has the property* P.

Note that while it does not make sense to ask whether a question is true in a structure, it makes a perfect sense to ask whether a question is resolved by an information state. The semantic support clauses for questions specify under what conditions are questions resolved:

- $s \vDash_e \varphi \mathbin{\vee\mkern-13mu\vee} \psi$ iff $s \vDash_e \varphi$ or $s \vDash_e \psi$,
- $s \vDash_e \exists\mkern-11mu\exists x\varphi$ iff for some $o \in U$, $s \vDash_{e(o/x)} \varphi$.

This looks like the usual clauses for disjunction and existential quantifier but note that the defined symbols $\vee$ and $\exists$ behave differently in the information-based semantics. For example, the difference between $\exists\mkern-11mu\exists$ and $\exists$ is illustrated when we spell out the semantic clauses:

- $s \vDash_e \exists\mkern-11mu\exists xPx$ iff there is $o \in U$ s.t. for every $\mathcal{M} \in s$, Px is true in $\mathcal{M}$ w.r.t. $e(o/x)$,
- $s \vDash_e \exists xPx$ iff for every $\mathcal{M} \in s$ there is $o \in U$ s.t. Px is true in $\mathcal{M}$ w.r.t. $e(o/x)$.

We can define first-order inquisitive logic as the set of formulas that are supported by every information state. Despite some serious effort to resolve this problem, it is still an open question whether inquisitive logic is recursively axiomatizable. It is also not known whether the related consequence relation is compact. In my talk these central problems of inquisitive semantics will be discussed together with some positive results obtained in the area (for example completeness results for various fragments of the language). I will also present an algebraic approach to these issues based on (Punčochář, 2021) that I believe might be helpful in the solution of the main problems.

References

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