From Semantic Games to Analytic Calculi

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It is well known that Tarski's notion of truth in a model can be characterized in terms of a game between a *Verifier* (or *Proponent*) and a *Falsifier* (or *Opponent*). Jaakko Hintikka referred to this fact as *game theoretic semantics* and proposed a generalization of the semantic game for classical first-order logic featuring imperfect information, leading to Independence Friendly (IF) logic. Here, however, we will look at another application of semantics games. Starting with the simplest semantic game, namely that for propositional classical logic, we will show how a systemic search for winning strategies for *Proponent* corresponds to Gentzen's sequent calculus for classical logic if we abstract away from concrete models. The central ingredient in this transformation of games into proof systems consists in lifting individual states of the semantic game to so-called disjunctive states, where all possible moves of *Proponent* are recorded. It turns out that classical sequents can be viewed as representations of disjunctive states and that Gentzen's logical rules directly correspond to Hintikka's rules for the semantic game.

The interest in the indicated method of lifting states to disjunctive states and then mapping those disjunctive states into objects of inference in an appropriate calculus, lies in its flexibility. Starting, e.g., with many-valued models or with Kripke models for modal logics, one may turn corresponding semantic games into various forms of analytic calculi. The term 'analytic' can be read in two (related) ways, here. The resulting calculi are analytic in the sense of avoiding the cut-rule. But they are also analytic in the wider sense of relating inference rules to the intended semantics in a systematic manner. The paradigmatic case for this method – revisited in some details in this presentation – is that of propositional Łukasiewiz logic, where one transforms Giles's semantic game into a corresponding hypersequent calculus.

The purpose of this talk is not to present new results or technical details that are needed to prove corresponding adequateness theorems, but rather to provide a gentle introduction, guided by examples, that is intended to make the overall methodology accessible without appealing to specific background knowledge. It also allows us to advertise an ongoing research project.