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Is nonlinearity relevant for detecting changes in EEG ?

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Summary

Transient changes of dynamical complexity in preictal, ictal and interictal invasive EEG recordings of an epileptic patient were characterized by linear and nonlinear measures related to entropy rates, as well as by correlation dimension and a nonlinearity index. All measures brought qualitatively equivalent results, however, the nonlinear coarse-grained entropy rate was found to be the most sensitive measure.

Keywords

epilepsy, invasive EEG, nonlinear analysis, entropy rate, correlation dimension

1 Introduction

For almost two decades there has been a sustained interest in describing and analysing the "brain dynamics" and its measurable effects such as the EEG within the framework of nonlinear dynamics and theory of deterministic chaos. Numerical tools from the latter theory were introduced into the EEG analysis with belief that standard linear analysis missed important information, provided the EEG was a nonlinear, possibly chaotic process. Several recent thorough studies have confirmed a nonlinear component in the EEG dynamics, however, signatures of low-dimensional chaos were not found (Paluš 1996a, 1999 and references therein). These results pose the question about adequacy of applying so called chaotic measures (dimensions, Lyapunov exponents) in the EEG analysis. In a couple of studies (Paluš 1998, 1999) it is shown that the chaotic measures applied to stochastic or noisy chaotic data do not bring different information than standard statistical analyses. In particular, in a series of numerical experiments (Paluš 1998), Gaussian random deviates were added to a set of chaotic time series with different Lyapunov exponents. It was demonstrated that the estimated Lyapunov exponents failed to distinguish different noisy chaotic time series when relatively small scales were used. The distinction was reestablished by using larger scales. Using larger scales, however, the estimated Lyapunov exponents were determined by macroscopic statistical properties of analysed series and provided the same information as the autocorrelation function and/or coarse-grained mutual information. On the other hand, states of chaotic systems can be discernible by using an entropy rate defined for linear Gaussian processes and computed from spectral densities (Paluš 1997). It seems that an entropy rate (or its coarse-grained version) is the property which allows classification of complex dynamical processes, irrespectively of their origin (linear or nonlinear, deterministic – chaotic or stochastic). In this paper we demonstrate application of such entropy rates in analysis of an epileptic EEG and compare linear and nonlinear versions of this measure, as well as its results with results of a correlation dimension algorithm.

2 Entropy rates

Consider a complex, dynamic process evolving in time. A series of measurements done on such a system in consecutive instants of time $t = 1, 2, \ldots$ is usually called a time series $\{y(t)\}$. Consider further that the temporal evolution of the studied system is not completely random, i.e., that the state of the system in time t in some way depends on the state in which the system was in time $t - \tau$. The strength of such a dependence per a unit time delay τ, or, inversely, a rate at which the system "forgets" information about its previous states, can be an important quantitative characterization of temporal complexity in the system's evolution. The time series $\{y(t)\}$, which is a recording of (a part of) the system temporal evolution, can be considered as a realization of a stochastic process, i.e., a sequence of stochastic variables. Uncertainty in a stochastic variable is measured by its *entropy*. The rate in which the stochastic process "produces" uncertainty is measured by its *entropy rate*. The concept of entropy rates is common to the theory of stochastic processes as well as to the information theory where the entropy rates are used to characterize information production by information sources (Cover and Thomas 1991).

Alternatively, the time series $\{y(t)\}$ can be considered as a projection of a trajectory of a dynamical system, evolving in some measurable state space. A. N. Kolmogorov (1959), who introduced the theoretical concept of classification of dynamical systems by information rates, was inspired by the information theory and generalized the notion of the entropy of an information source. The Kolmogorov-Sinai entropy (KSE thereafter) (Kolmogorov 1959, Sinai 1976) is a topological invariant, suitable for classification of dynamical systems or their states, and is related to the sum of the system's positive Lyapunov exponents (LE) according to the theorem of Pesin (1977).

Thus, the concept of entropy rates is common to theories based on philosophically opposite assumptions (randomness vs. determinism) and is ideally applicable for characterization of complex biological processes, where possible deterministic rules are always accompanied by random influences. What is the role of nonlinearity in such a characterization? Of course, KSE is positive only for chaotic processes, which are by nature nonlinear. That is, applying KSE for characterization of time series, nonlinear character of an underlying process is implicitly considered. On the other hand, considering stochastic processes, the entropy rate is defined for a general stationary process, linear or nonlinear.

Possibilities to compute the entropy rates from data are limited to a few exceptional cases: for stochastic processes it is possible, e.g., for finite-state Markov chains (Cover and Thomas 1991). Another possibility, which will be considered here, is the case of linear stochastic – Gaussian processes: If $\{X_i\}$ is a zero-mean stationary Gaussian process with spectral density function $f(\omega)$, its entropy rate h_G, apart from a constant term, can be expressed using $f(\omega)$ as (Ihara 1993, Anh and Lunney 1995):

$$h_G = \frac{1}{2\pi} \int_{-\pi}^{\pi} \log f(\omega) d\omega. \tag{1}$$

Dynamics of a stationary Gaussian process is fully described by its spectrum. Therefore the estimation of the entropy rate of a Gaussian process (GPER thereafter) is reduced to estimation of its spectrum.

In the case of dynamical systems, Fraser (1989) proposed to estimate KSE from the asymptotic behavior of marginal redundancy, computed from a time series generated by the dynamical system. The marginal redundancy $\varrho^n(\tau)$ (see Paluš 1995, 1996a,b,c and references therein), for $n = 2$ also known as mutual information, measures the amount of information about a system in time $t + (n-1)\tau$ contained in measurements done consecutively in times t, $t + \tau$, ..., $t + (n-2)\tau$. The asymptotic behaviour of $\varrho^n(\tau)$, however, is practically impossible to attain in experimental situations when time series of limited length and precision and possibly with noise are only available. Therefore Paluš (1996b) has proposed to give up the effort for estimating the exact entropy rates, and to define "coarse-grained entropy rates" (CER's) instead. The CER's are not meant as estimates of the exact entropy rates, but rather as relative measures of regularity and predictability of analysed time series.

In a particular application, we compute the marginal redundancies $\varrho^n(\tau)$ for all analyzed datasets and find such τ_{max} that for $\tau' \geq \tau_{max}$: $\varrho^n(\tau') \approx 0$ for all the datasets. Then we define a norm of the marginal redundancy

$$||\varrho^n|| = \frac{\sum_{\tau=\tau_0}^{\tau_{max}} \varrho^n(\tau)}{\tau_{max} - \tau_0}. \tag{2}$$

Having defined the norm $||\varrho^n||$, the difference

$$h^{(0)} = \varrho^n(\tau_0) - ||\varrho^n||, \tag{3}$$

can be considered as the definition of CER. Or, in some application the normalized quantity

$$h^{(1)} = \frac{\varrho^n(\tau_0) - ||\varrho^n||}{||\varrho^n||} \tag{4}$$

can be considered as an alternative definition of CER. The ability of the CER's to discern systems with different exact entropy rates, in particular, to discern different states of a chaotic system, is illustrated in Fig. 1. For the well-known chaotic baker map (Farmer et al. 1983), its KSE (or, equivalently, the positive Lyapunov exponent, LE) can be expressed analytically as a function of the system's parameter α (Fig. 1a). Then time series for different values of α were generated and their CER's were computed and plotted as functions of the parameter α (Fig. 1 b). The CER $h^{(1)}$ provides distinction and classification of the states (with different α) of the baker system equivalently to the analytically computed LE/KSE, i.e., the baker system's exact entropy rate. We can see that, in the case of entropy rates, both the coarse-grained or macroscopic version of a microscopic (defined via limits) quantity, and the microscopic quantity itself provide equivalent classification of dynamical processes. Similarly, the states of the chaotic baker system are distinguished and correctly classified by a formal application of the linear entropy rate h_G (1), defined for Gaussian processes. This GPER h_G is in fact microscopic, but linear measure (Paluš 1997).

3 Material and methods

The analysed data were recordings from subdural strips implanted to a 18-year female patient, who has been suffering from complex partial seizures from the age of 7 years. Using the EEG/ECoG video monitoring, a temporal ictal focus was localized. Here we use the recordings from the electrode closest to the focus. Two records were analysed in the presented example: 75 minutes of interictal recording and 28 minutes of preictal, followed by several minutes of ictal recording. Sampling rate was 200 Hz, the first part of processing was a wavelet based bandpass filter, which allowed to pass frequencies higher than 1Hz and lower than 50 Hz. The data were characterized by using CER (3), GPER (1), correlation dimension computed by using the Grassberger-Proccacia (1983) algorithm modified according to Dvořák and Klaschka (1990) and a "nonlinearity index" computed as the L_2-difference between mutual information of the EEG signal and its isospectral surrogate data (Theiler et al. 1992, Paluš 1995). The latter were generated as realizations of a linear stochastic process with the same spectrum as the analyzed segment of the EEG data. For comparison, CER's were also computed from surrogate data. All the quantities were computed in a window of 2048 samples, which was moved over the whole series by step of 1024 samples. CER and the nonlinearity index were computed using the marginal redundancy of order two, i.e., the mutual information, using the time lags 1 – 50 samples. The mutual information was estimated using 8 marginal equiquantal bins (see Paluš 1995, 1996b). The results were plotted as functions of time from the beginning of the record. All the quantities were plotted in a relative scale given as difference from a mean value (of the particular quantity) obtained from the interictal recording, divided by the standard deviation (SD), obtained also from the interictal recording.

4 Results

The coarse-grained entropy rate (CER) "profile" for the preictal and ictal segment is plotted in Fig. 2a. The beginning of the seizure at the relative time 23.40 minutes is clearly recognisable by the sudden CER decrease to the lowest values (under -10 SD). In the relative times 1.1, 3.7, 6.1, 10.2 and 17.3 minutes there are sudden, highly statistically significant decreases of CER. In the interictal recording (Fig. 2b) there are also several significant CER decreases, however, they are lower in depths, shorter in duration and less frequent in their occurrence in comparison with the preictal record. Decreases of dimensional complexities in preictal EEG recordings have recently been identified as features predicting epileptic seizures (Lehnertz and Elger 1998, Martinerie et al. 1998). Can a sequence of relatively deep and frequent CER (complexity) decreases be considered as a precursor for a seizure? Until now we have processed only two additional preictal recordings, though with similar results, yet we cannot assess predictive value of these changes in the EEG complexity, here measured by CER. Therefore, we will only discuss ability of various measures to detected these changes, especially from the viewpoint of linearity vs. nonlinearity.

At the first step, however, we compare the CER results with more familiar nonlinear measure of "complexity", namely the correlation dimension (CD thereafter, Figs. 2c,d). Qualitatively the results are very similar to those obtained by CER, however, the CD decreases are less sharp than the CER decreases, i.e., CER seems more sensitive in this case. We cannot conclude, however, that CER is more sensitive than CD in general, because different algorithms for CD can have different properties. At this point we consider as important the fact, that the CD results are not qualitatively different from the CER results.

The preictal-ictal CER profile from Fig. 2a is again plotted in Fig. 3a. Here it can be compared with results obtained using the linear entropy rate GPER (Fig. 3b). In the qualitative sense the linear entropy rate GPER provides the same profile as (nonlinear) CER, i.e., the changes of complexity of the EEG dynamics are also detectable by linear tools. The sensitivity of GPER, however, is considerably lower than the sensitivity of CER. Applying the nonlinearity index (Fig. 3d) we can see that the segments of increased "level of nonlinearity" coincide with the decreases of entropy rates. Thus, one could hypothesize that the lower GPER sensitivity is due to a nonlinear character of the EEG dynamics. This is, however, only partially true. Different measures have also different numerical properties. In order to obtain a fair comparison of "amounts of information" obtainable on linear versus nonlinear levels of description, we compute the same CER as in Fig. 3a, but from linear stochastic surrogate data of the original EEG in each characterized segment. The surrogates replicate linear properties of analysed segments (spectrum, autocorrelation, histogram), while nonlinear structures, present in the original data, are not transferred into the surrogates. We can see in Fig. 3c that the sensitivity of the surrogate CER is somewhere between the sensitivity of GPER and CER obtained from the EEG data. So the reasons of the lower sensitivity of the linear entropy rate GPER are twofold: First, the segments of decreased dynamical complexity are "more nonlinear" than the segments of normal background activity. Therefore linear measures indeed loose some information. The second reason is given by numerical properties of particular measures. The linear GPER is derived for continuous processes and reflects all (linear) data properties, including noise. CER, as well as other nonlinear measures, are usually confined to some typical scale, or range of scales, so they effectively filter data. This filtering increases sensitivity to dynamical changes, since influence of background or noise effects are diminished, provided the scales were chosen adequately to the target dynamics.

5 Discussion and Conclusion

We have found transient significant changes in the dynamical complexity of a preictal subdural EEG recordings of an epileptic patient. Due to a limited amount of processed data we cannot yet assess a value of these changes for seizure prediction. We concentrated on studying ability of various measures to detect such changes. Using two types of entropy rates, we have found that the changes were detectable on both linear (GPER) and nonlinear (CER) levels. These results confirmed previous numerical studies which suggested that a linear entropy rate GPER, computed from estimated spectrograms, could distinguish different states of nonlinear chaotic processes. So that the linear approaches, namely the advanced ones, such as the entropy rate GPER, should not be underestimated. On the other hand, the sensitivity to dynamical changes of the linear measure (GPER) has been found lower than the sensitivity of the nonlinear measure (CER). This lower sensitivity could be critical when analysing recordings from electrodes not so close to an epileptic focus, or scalp recordings. Therefore nonlinear approaches should play an important role in detecting changes in the EEG. The question is, however, the choice of appropriate nonlinear measures. Interestingly enough, in our study the correlation dimension brings qualitatively the same information as the coarse-grained entropy rates. Considering that there is probably no low-dimensional attractor underlying the EEG, the real dynamical changes are those of the signal entropy rate, which apparently influences formal dimensional estimates. It is, however, the question of further study whether the entropy rate changes can lead to a reliable seizure prediction, or we should concentrate on different properties.

As a final note we stress that this study was concentrated on characterizing individual EEG channels. A challenging field for nonlinear approaches is analysis of relations between/among various channels in order to find changes in coordination or synchronization on various levels.

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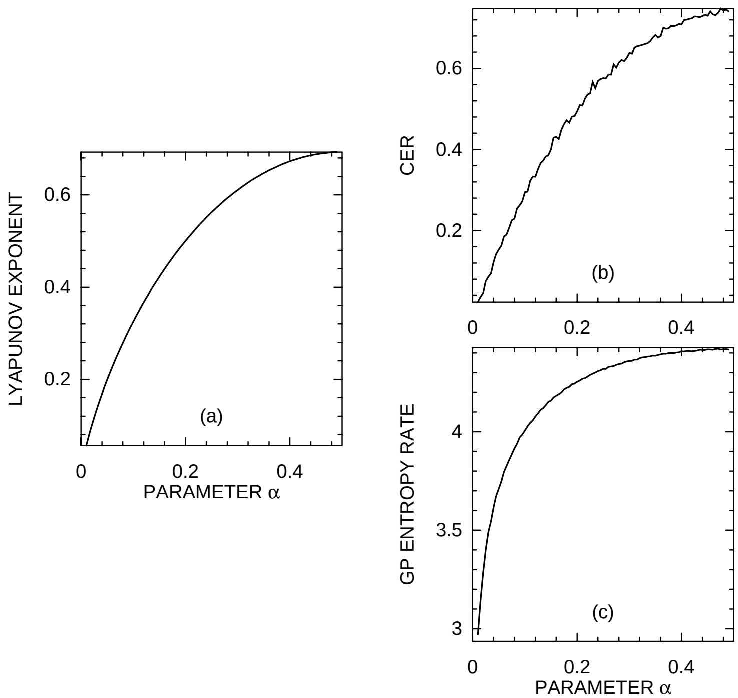

Figure 1: *(a) Positive Lyapunov exponent (or Kolmogorov-Sinai entropy) of the chaotic baker map computed as the analytical function of the parameter α. (b) Coarse-grained entropy rate $h^{(1)}$ estimated from time series generated by the baker system for 97 different values of the parameter α. (c) Linear entropy rate GPER estimated from time series (via spectrogram) generated by the baker system for 97 different values of the parameter α.*

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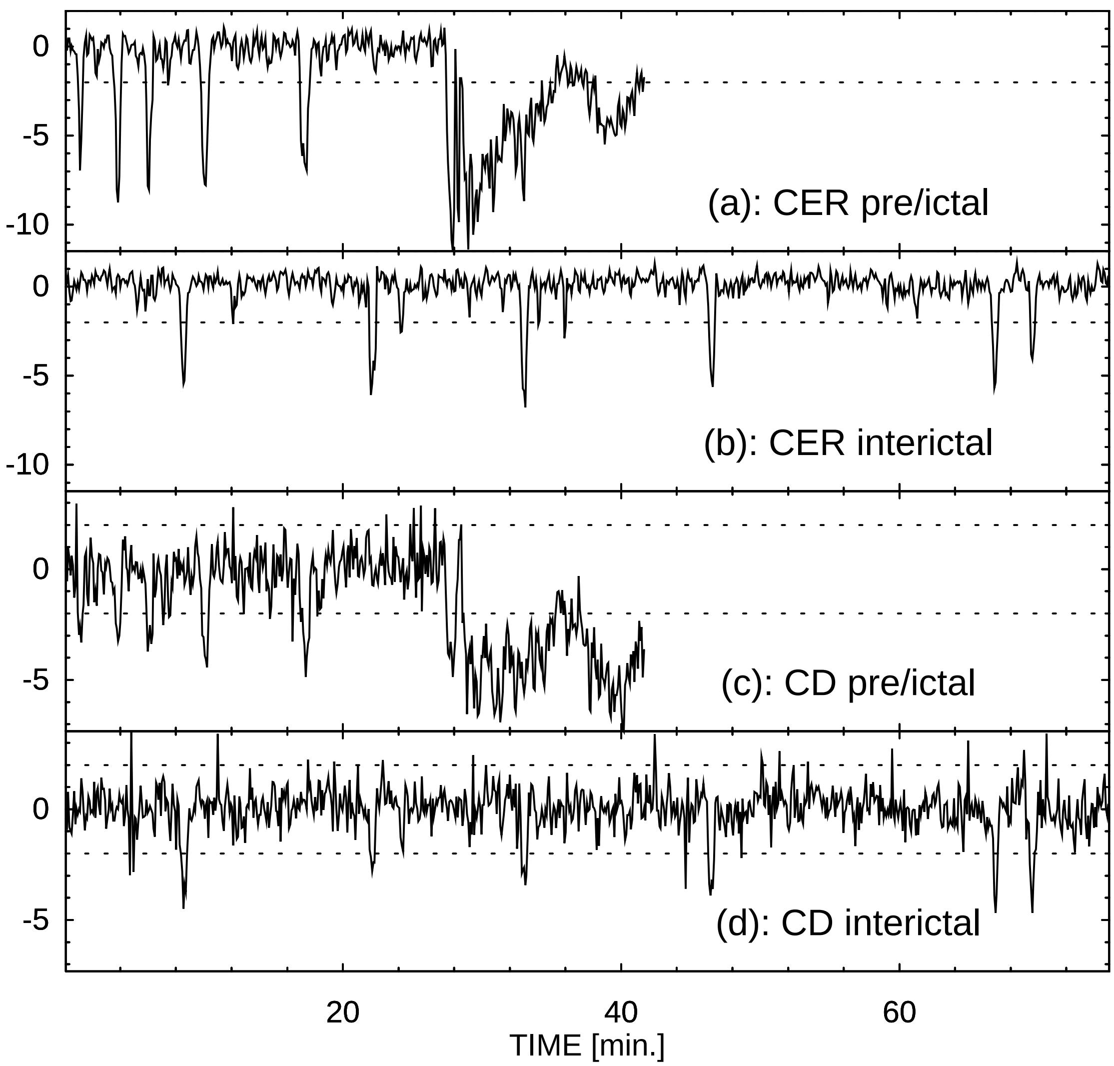

Figure 2: *Coarse-grained entropy rate $h^{(0)}$ (a, b) and correlation dimension (c, d) profiles for preictal and ictal (a, c) and interictal (b, d) EEG recordings. All measures are plotted in a relative scale given as the difference from an interictal mean in the number of interictal standard deviations (SD). The dashed vertical lines (in some cases coinciding with upper frame lines) give the interictal "normal" range -2, 2 SD, i.e., decreases under -2 are statistically significant.*

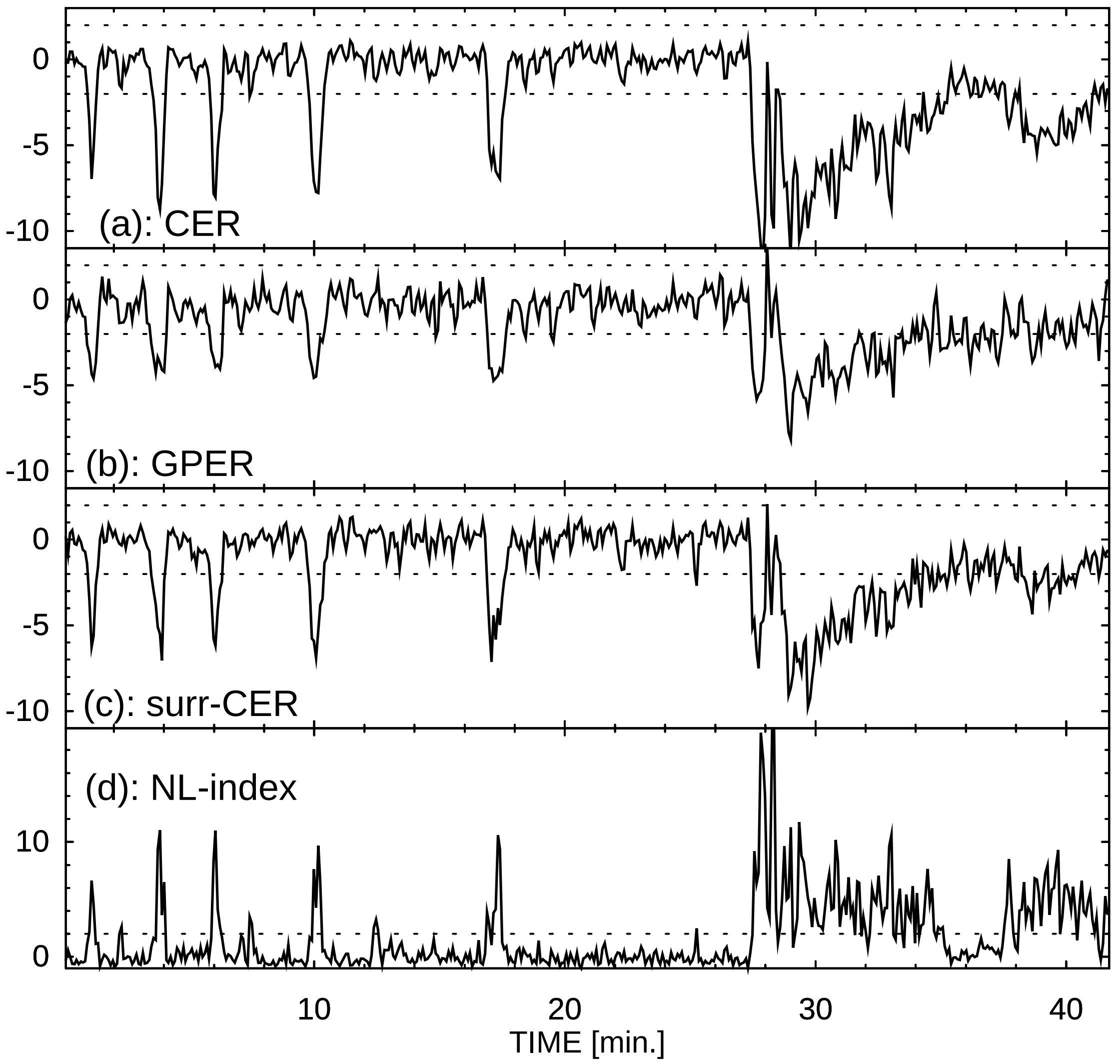

Figure 3: *Profiles for the preictal and ictal EEG recordings obtained from (a) coarse-grained entropy rate* $h^{(0)}$*, (b) linear entropy rate GPER, (c) coarse-grained entropy rate* $h^{(0)}$ *computed from linear surrogate data, and (d) nonlinearity index. For explanation of ordinate scales see the legend of Fig. 2.*